RYERSON POLYTECHNIC UNIVERSITY

DEPARTMENT
OF
MATHEMATICS, PHYSICS AND COMPUTER SCIENCE

MTH405 — Term Test 1
February 9, 2001

Time allowed: 60 **minutes.** **Name (print):** ______________________

Total marks: 50 **Student Number:** ______________________

Instructions:

- Please keep your Ryerson photo ID card displayed on your desk during the test.
- Electronic devices such as calculators, cell-phones, pagers and Walkmen must be turned off and kept inaccessible during the test.
- Verify that your paper contains 9 questions and 5 pages. The last page should have no questions on it.
- Please write only in this pamphlet. Use of scrap paper or additional enclosures is not allowed. If you need more space, continue on the back of the page, directing the marker where the solution continues with a bold sign.
- You must show all your work. A correct answer alone may be worth nothing.

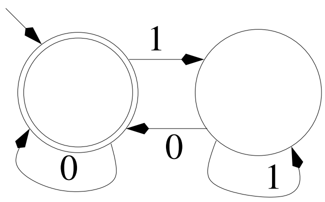
[4 Mk] 1. Give an example of a language over $\{0, 1\}$ which no DFA can recognize.

[5 Mk] 2. Give a regular expression denoting the set L of all strings over $\{a, b, c\}$ which fail to contain occurrences of both a and c. That is $L = \left\{x \in \{a, b, c\}^* \;\middle|\; |x|_a \cdot |x|_c = 0\right\}$.

4 Mk

3. Give the graph for an **FST** which inverts all the bits in any binary string. Thus for example on input of 10011110 it would give as output 01100001.

4. Describe in words, as succinctly as you can, the language recognized by the following FSA.



5 Mk

5 Mk

5. Give the graph for a DFA which recognizes the language of all strings over $\{0, 1\}$ which have differing first and last characters. Thus, for example, 110110 would be in this language while 010110 would not.

6. Describe in words, as succinctly as you can, the language denoted by the regular expression

$$(0^*10^*)^*.$$

5 Mk

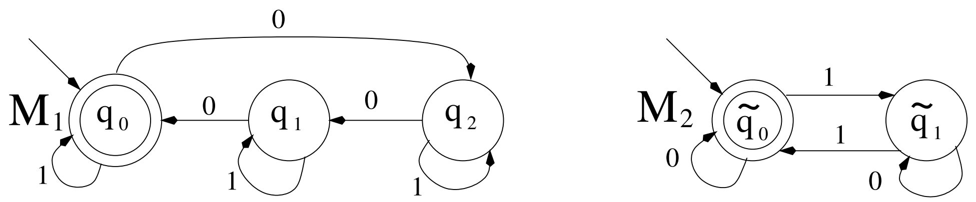
6 Mk

7. Use Warshall's method to find the transitive closure of the following binary relation R on the set $\{a, b, c, d, e\}$

$$R = \{(b, a), (a, b), (b, e), (d, a), (b, c), (e, a)\}.$$

6 Mk

8. Give the graph for a deterministic finite state automaton M such that $L(M) = L(M_1) \cap L(M_2)$ by forming a "product" of the DFAs M_1 and M_2 below.



9. In this problem you must not use previously learned intuitive notions or definitions of complete binary trees (CBTs for short), but must use exclusively their definition by structural induction given here.

   We define what it means to say T is a **complete** binary tree (over a given set A) of height $h(T)$ having a set $N(T)$ of $n(T)$ nodes thus:

   Base If we are given any $r \in A$ then $T = (r, \bullet, \bullet)$ is a CBT over A with:

   (a) $N(T) = \{r\}$,
   (b) $n(T) = 1$,
   (c) $h(T) = 0$.

   Induction Step If we are given any $r \in A$ and any two complete binary trees T_1 and T_2 over A such that $N(T_1) \cap N(T_2) = \emptyset$ and $r \notin N(T_1) \cup N(T_2)$ **and** $h(T_1) = h(T_2)$ then $T = (r, T_1, T_2)$ is a CBT over A with:

   (i) $N(T) = \{r\} \cup N(T_1) \cup N(T_2)$,
   (ii) $n(T) = 1 + n(T_1) + n(T_2)$,
   (iii) $h(T) = 1 + h(T_1) = 1 + h(T_2)$.

   Prove by structural induction that $P(T)$ holds for every complete binary tree T, where

   $$P(T) \text{ denotes the statement } 1 + n(T) = 2^{1+h(T)}.$$

   The marker will be paying great attention to the form and the logical correctness and clarity of your proof. For any credit you must use **only** structural induction, you must **not** use weak or strong mathematical induction or other methods. If you need more space, continue your solution on the blank page attached.

10 Mk

THIS IS A BLANK PAGE FOR CONTINUATION OF YOUR SOLUTION TO QUESTION 9