RYERSON POLYTECHNIC UNIVERSITY

DEPARTMENT
OF
MATHEMATICS, PHYSICS AND COMPUTER SCIENCE

MTH405 — Term Test 2
March 24, 2000

Time allowed: 50 minutes. **Name (print):** ____________________

Total marks: 40 **Student Number:** ____________________

Instructions:

- Please keep your Ryerson photo ID card displayed on your desk during the test.
- Electronic devices such as calculators, cell-phones, pagers and Walkmen must be turned off and kept inaccessible during the test.
- Verify that your paper contains 8 questions and 6 pages.
- Please write only in this pamphlet. Use of scrap paper or additional enclosures is not allowed. If you need more space, continue on the back of the page, directing the marker where the solution continues with a bold sign.
- You must show all your work. A correct answer alone may be worth nothing.

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1. Using the method described in the text and in lectures to build up an NFA for a regular expression by construction rules corresponding to the form of the regular expression, construct an NFA whose language is $(aa)^*$. Do not simplify your machine at any stage of its construction.

3 Mk

2. Let M denote the following NFA:

Current State	Current Accept?	Next state when input is: a	b	c	ε
q_0	NO	$\{q_1, q_2\}$	$\varnothing$	$\{q_0\}$	$\{q_1\}$
q_1	YES	$\varnothing$	$\{q_1\}$	$\varnothing$	$\{q_2\}$
q_2	NO	$\{q_1\}$	$\{q_1\}$	$\varnothing$	$\varnothing$

3 Mk

(a) Draw a tree showing all possible computational paths for the string *cab* on the NFA M. State whether or not this string is accepted and why.

5 Mk

(b) Using the method described in lectures, create an equivalent FSA (graph and table) for the NFA M above.

3. Consider the following context free grammar G:

$$S \to ASB|\varepsilon$$

$$A \to a|b$$

$$B \to c|d$$

2 Mk

(a) Give a derivation tree for the string $bbdc \in L(G)$ relative to the grammar G.

2 Mk

(b) Give a leftmost derivation sequence for the string $bbdc \in L(G)$ relative to the grammar G.

2 Mk

(c) Use words to succinctly describe the language generated by the grammar G.

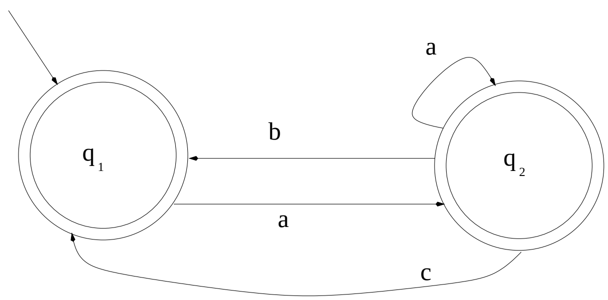
3 Mk

4. Give a **context-free** grammar for the language $L = \{a^m b^n | m, n \in \mathbb{N}, m < n\}$.

5. Use the method employing transforming GNFAs to obtain a regular expression for the language recognized by the NFA shown below.

   - In the interests of uniform marking, please eliminate the nodes in your GNFA in the order of their subscripts.
   - Please simplify the arc labels for each new state graph you draw.
   - Use two extra colours on your state graphs as directed in class (ask if unsure).

6 Mk



6. There is only one major flaw in the "proof" of the claim given below. You are to find it. Place a ★ symbol beside the first statement which is mathematically incorrect. Then explain, using at most two sentences, the nature of the error that has been made. Don't guess as marks may be deducted for wrong answers.

3 Mk

Claim: `There are no finite, non-empty, regular languages.`

- Let us suppose the claim above were false.
  (We will derive a contradiction.)
- There would then exist some non-empty, regular language L say, containing only finitely many strings.
- We could then select from L a string w whose length was maximum for L.
- The pumping lemma would apply to L.
- Since $w \in L$, by the pumping lemma, $w_2 \in L$ also.
- But w_2 would be longer than w contradicting the fact that w had maximum length in L.
- By the principle of proof by contradiction the claim must be true.

7. Give a **regular** grammar for the language L of all strings over the alphabet $\{a, b\}$ which contain at least one occurrence of the symbol a.

4 Mk

8. The pumping lemma may be used to show each of the three languages below is NOT regular. Using the letter **p** to denote the pumping constant, in each case give a string $w = w_1$ and an $i \in \mathbb{N}$ such that the resulting pumped string w_i contradicts the assertion of the pumping lemma.

7 Mk

(a)

$$L = \left\{ x \in \{a, b\}^* \;\middle|\; |x|_a < |x|_b \right\}$$

My choice for w would be:
My choice for i would be:

(b)

$$L = \left\{ a^m b^n \;\middle|\; m, n \in \mathbb{N}, m > n \right\}$$

My choice for w would be:
My choice for i would be:

(c)

$$L = \{a^{(2^n)} | n \in \mathbb{N}\}$$

My choice for w would be:
My choice for i would be: