

RYERSON POLYTECHNIC UNIVERSITY
DEPARTMENT
OF
MATHEMATICS, PHYSICS AND COMPUTER SCIENCE

MTH405 — Final Exam
April 25, 2000

Family Name (print): _____ Given Name (print): _____

Signature: _____ Student Number: _____

Instructions:

Time allowed: 3 hours.

- Verify that your paper contains 14 questions and 9 pages.
- Please keep your Ryerson photo ID card displayed on your desk during the exam.
- Aids are **not** allowed.
- Electronic devices such as calculators, cell-phones, pagers and Walkmen must be turned off and kept inaccessible during the test.
- You must show all your work. A correct answer alone may be worth nothing.
- Please write only in this pamphlet. Use of scrap paper or additional enclosures is not allowed. If you need more space, continue on the back of the page, directing the marker where the solution continues with a bold sign.
- In this exam paper the symbol \mathbb{N} denotes the set $\{0, 1, 2, 3, \dots\}$.

1. Give the state graph for a (deterministic) finite state automaton which recognizes the language of all strings over $\{a, b, c\}$ in which the number of a 's plus the number of b 's is an even integer.

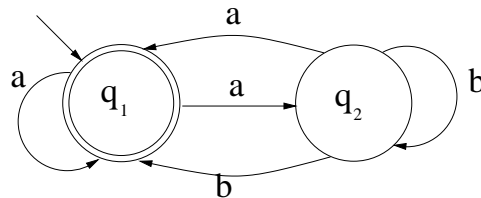
4 %

2. An FSA has input alphabet $\Sigma = \{a_1, \dots, a_m\}$ and set of states $Q = \{q_0, \dots, q_{n-1}\}$. How many arcs are there in its state diagram?

2 %

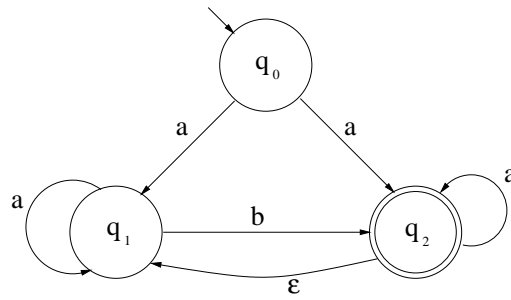
3. Using the method employing GNFA's described in lectures, convert the following NFA to an equivalent regular expression.

- In the interests of uniform marking, please eliminate the nodes in your GNFA's in the order of their subscripts.
- Please simplify the arc labels for each new state graph you draw.
- Use colour in your state graphs as directed in class (ask if unsure).
- It is not necessary to express your final answer in the simplest form, in order to earn most of the marks.



10 %

4. Let M denote the following NFA:



(a) Draw a tree showing **all** possible computational paths for the string ab when it is processed by M . State whether or not this string is accepted and why.

3 %

(b) Give the state table for M .

3 %

(c) Using the method described in lectures, create (table and graph) an equivalent FSA for the NFA M given above.

5 %

(d) Using words, describe as succinctly as possible, the language recognized by the FSA you found in part 4c) above.

3 %

5. State Church's Thesis. What evidence is there for it?

4 %

6. Give an example of a statement that is usually proved by "diagonalization".

2 %

7. Consider the language L denoted by the regular expression $(a \vee ba \vee bb)(a \vee b)^*$.

2 %

(a) Give two strings which **are** in L .

4 %

(b) Give two strings from $\{a, b\}^*$ which **are not** in L .

8. Consider the language L over the alphabet $\{a, b\}$ consisting of all non-empty strings that begin and end with the same symbol.

4 % (a) Give a regular expression which denotes the language L .

6 % (b) Give an NFA which recognizes the given language L and has 4 or fewer states.

5 % (c) Give a regular grammar which generates the language L .

6 % (d) Give a PDA which recognizes the given language L and has 3 or fewer states.

4 % 9. Give a context-free grammar for the language $\{x \in \{0, 1\}^* \mid x = x^R\}$ of all palindromes over the alphabet $\{0, 1\}$.

10. Draw a Venn diagram illustrating the containment relationships among the following classes of languages over a given alphabet Σ : Finite, decidable, context free, regular, recursively enumerable, all languages over Σ .

If you can think of one, show on your diagram, for each class, an example of a language that is in that class but not in the next smaller class.

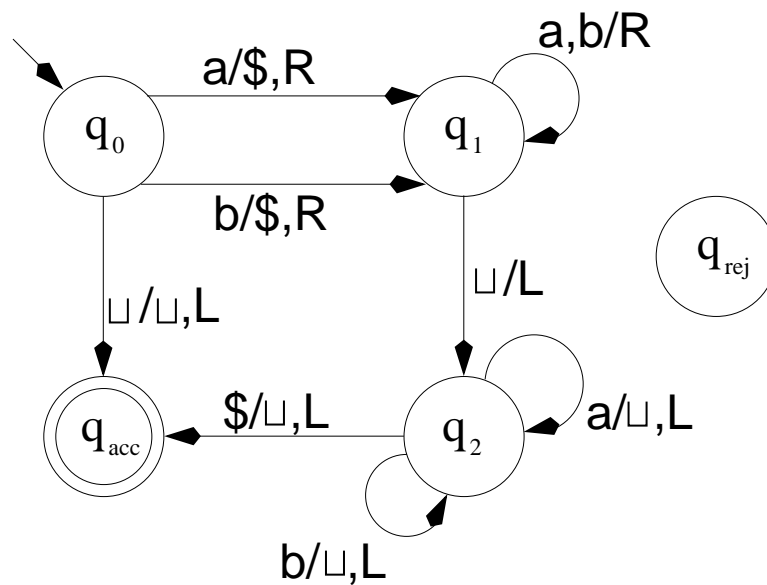
For each class for which the issue was discussed in lectures, name the types of grammars and computational devices that are associated with these languages; also give any synonyms you know for each language type.

6 %

11. Give the state graph for a (deterministic) Turing machine which recognizes the language over the input alphabet $\Sigma = \{a, b\}$ consisting of all strings which contain no occurrences of a . Your machine should have 4 or fewer states.

6 %

12. Consider the following Turing machine where $\Sigma = \{a, b\}$, $\Gamma = \{a, b, \$, \sqcup\}$, and where \sqcup is used to represent the blank tape symbol. To reduce clutter all arcs going to q_{rej} have been suppressed.



Give the sequence of “snapshots” of the configurations this TM goes through when started on the string aba .

5 %

13. Suppose that a_0, a_1, a_2, \dots is a sequence defined as follows

$$a_i = \begin{cases} 0 & \text{if } i = 0, \\ 1 & \text{if } i = 1, \\ a_{i-1} + a_{i-2} & \text{if } i \geq 2. \end{cases}$$

8 % Prove by strong mathematical induction that $a_i < 2^i$ for all $i \in \mathbb{N}$.

14. Use the pumping lemma to show that $L = \{a^m b^n c^{\min(m,n)} \mid m, n \in \mathbb{N}\}$ is not regular.

8 %