1. Consider the context free language $L = \{a^m b^n c^\ell d^m \mid \ell, m, n \in \mathbb{N}\}$.

7 mk

(a) Give a context free grammar for the language L.

$S \to aSd \mid \varepsilon \mid T$

$T \to bT \mid Tc \mid \varepsilon$

(b) Relative to the grammar you gave in part (a) give a left-most derivation for the string *aabbbcdd*.

$S \Rightarrow aSd$

$\Rightarrow aaSdd$

$\Rightarrow aaTdd$

$\Rightarrow aabTdd$

$\Rightarrow aabbTdd$

$\Rightarrow aabbbTdd$

$\Rightarrow aabbbTcdd$

$\Rightarrow aabbbcdd$

WOS MTM405 TEST 2 SOLUTIONS

*Student Number:*_____

2. Describe succinctly in English the language recognized by the PDA shown below

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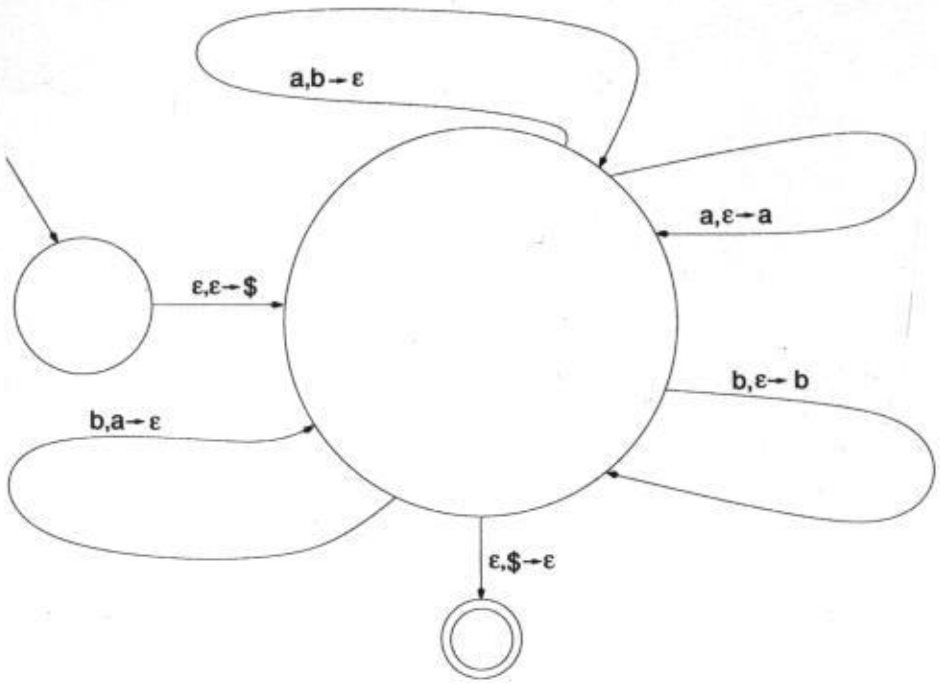

All strings over {a, b} which have equal numbers of a's and b's.

3. For the Turing machine below:

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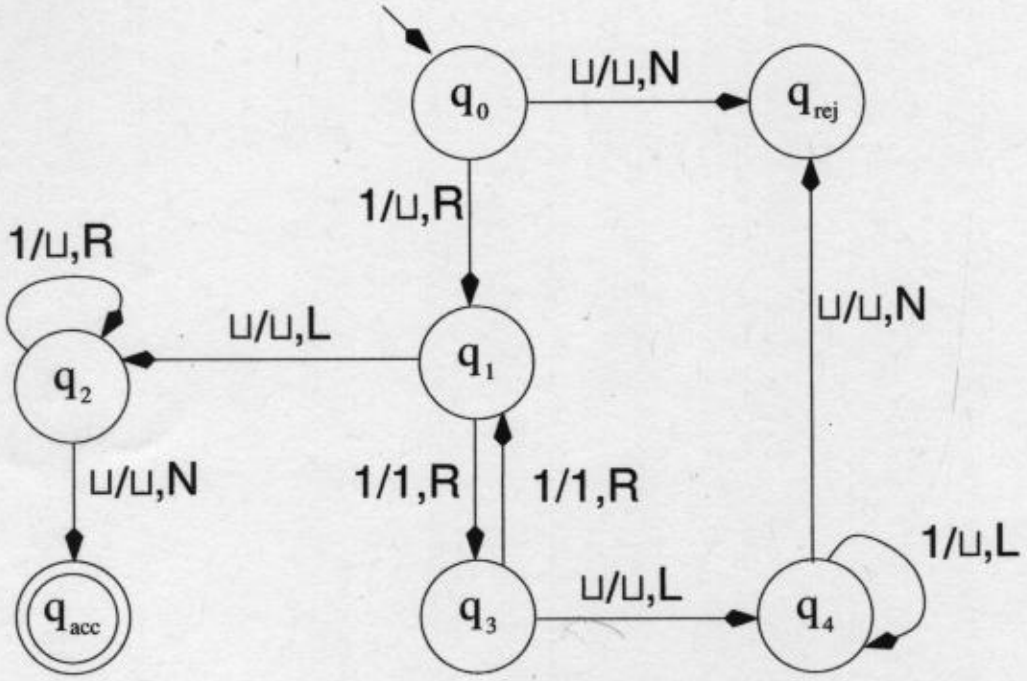

(a) List the configurations this machine passes through when started on the string 11.

$q_0 11 \vdash \sqcup q_1 1 \vdash \sqcup 1 q_3(\sqcup) \vdash \sqcup q_4 1 \vdash q_4(\sqcup) \vdash q_{rej}(\sqcup)$

(b) Describe in English exactly what this machine does when it is started with a string of 1s on its input tape.

An empty string is immediately rejected. The pair of states q_1/q_3 forms a flip/flop system. After 1^{2n+1} states q_1, after scanning 1^{2n} state is q_3. When first blank seen is when machine is in q_1 (input was 1^{2n+1}) the last 1 of input is erased & T.M goes to q_{rej}. When first blank seen is when machine is in q_3 (input was $1^{2s}, s>0$) the T. M. erases its tape, rewinds to left cell & rejects

4. The context free pumping lemma may be used to show each of the three languages below is NOT context free. Using the letter ℓ to denote the pumping constant, in each case give a string $w = w_1$ and an $i \in \mathbb{N}$ such that the resulting pumped string w_i contradicts the assertion of the context free pumping lemma. [6 mk]

(a)

$$L = \{x \in \{a, b, c\}^* \mid |x|_a = |x|_b = |x|_c\}$$

My choice for w would be:	$a^\ell b^\ell c^\ell$
My choice for i would be:	any natural number, except 1

(b)

$$L = \{uu \mid u \in \{a, b\}^*\}$$

My choice for w would be:	$a^\ell b^\ell a^\ell b^\ell$
My choice for i would be:	any natural number except 1

(c)

$$L = \left\{a^{(n^2)} b^n \mid n \in \mathbb{N}\right\}$$

My choice for w would be:	$a^{(\ell^2)} b^\ell$
My choice for i would be:	0 or 1 or 2

5. Using the "table" method described in lectures and labs, find the equivalent finite state machine to the one shown below which has the minimum nuber of states.

PREDECESSOR TABLE

current	PREVIOUS STATE IF INPUT WAS 0	PREVIOUS STATE IF INPUT WAS 1
q_0	ϕ	ϕ
q_1	ϕ	$\{q_0, q_1\}$
q_2	$\{q_1\}$	$\{q_3\}$
q_3	$\{q_0, q_2, q_3\}$	$\{q_2\}$

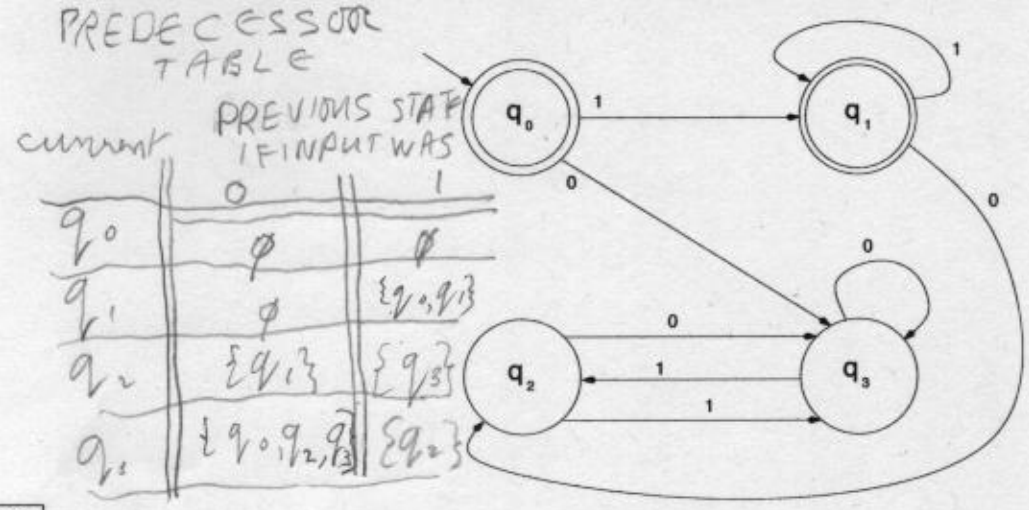

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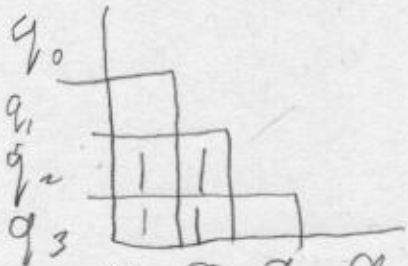
TABLE OF DISTINGUISHABLE STATE PAIRS



One pass thru this distinguishable q_0 q_1 q_2 q_3 pair finds that there no more derived pairs to be added.

From the table, the indistinguishable pairs are: $\{q_0, q_1\}$ and $\{q_2, q_3\}$.