MTH 207 Midterm Solutions

(1a) $\lim_{x\to 0} \frac{5+\sin(32x)}{7x^2-x-1} = \frac{5+\sin(32\cdot 0)}{7(0)^2-0-1} = 0.$

(1b) $\lim_{x\to 8^-} \frac{|x-8|}{x-8} = -1;\quad \lim_{x\to 8^+} \frac{|x-8|}{x-8} = 1.$ Therefore the limit does not exist.

(1c) $\lim_{x\to 0} \frac{1-\sqrt{1-x^2}}{x} = \lim_{x\to 0} \frac{1-\sqrt{1-x^2}}{x}\frac{1+\sqrt{1-x^2}}{1+\sqrt{1+x^2}} = \lim_{x\to 0} \frac{x}{1+\sqrt{1-x^2}} = 0$

(1d) $\lim_{x\to 0} \frac{\sin 4x}{x\tan(x+\pi/4)} = \lim_{x\to 0} 4\cdot\frac{\sin 4x}{4x}\cdot\frac{1}{\tan(x+\pi/4)} = 4\cdot 1\cdot\frac{1}{1} = 4$

(2) Want that $\lim_{x\to 1^-} f(x) = \lim_{x\to 1^+} f(x) = f(1)$ (clearly, $f(x)$ is continuous at all points $\neq 1$ because it is simply a polynomial there). $\lim_{x\to 1^-} f(x) = 2-c$, $\lim_{x\to 1^+} f(x) = 4-1 = 3$. So require that $2-c = 3 \to c = -1$.

(3) $f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h} = \lim_{x\to 0} \frac{\frac{1}{x+h+1}-\frac{1}{x+1}}{h} = \lim_{x\to 0} \frac{-h}{h(x+1)(x+h+1)} = \frac{-1}{(x+1)^2}.$

(4a) $y'(x) = \sec^2(\sqrt{1-x})(\frac{1}{2}(1-x)^{-1/2}(-1)).$

(4b) $y'(x) = -2x^{-3}\sin^2(x^3) + 2x^{-2}\sin(x^3)\cos(x^3)(3x^2).$

(4c) $y'(x) = \frac{(2\sqrt{x}+1)^2-(\sqrt{x})^2(2x^{-1/2}(2\sqrt{x}+1))}{(2\sqrt{x}+1)^4}.$

(5) Slope of tangent line at point x: $\tan' x = \sec^2 x$.
Want this slope to be $-(\frac{1}{-1/2}) = 2$. So solve $\sec^2 x = 2 \to \cos x = \pm 1/\sqrt{2} \to x = \pm\pi/4$.

(6a) $\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{1+\cos t}{2t+4} = 0 \;\to \cos t = -1 \;\to t = \pm\pi.$
So the points are; $t = \pi:\; (\pi^2-4\pi-1,\; -\pi),\quad t = -\pi:\; (\pi^2+4\pi-1,\; \pi)$.
Plot this curve in MAPLE to confirm;

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plot([t^2 + 4*t - 1, t + sin(t), t = -2*Pi..2*Pi]);
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(6b) $\frac{d^2y}{dx^2} = \frac{d}{dx}(\frac{dy}{dx}) = \frac{dy'/dt}{dx/dt} = \frac{(-\sin t)(2t+4)-(1+\cos t)2}{(2t+4)^3}.$
At $t = \pi: \frac{d^2y}{dx^2} = 0$, at $t = -\pi: \frac{d^2y}{dx^2} = 0$.

(7a) $\frac{dy}{dx} = \frac{-y}{x+2y}.$ **(7b)** $\frac{d^2y}{dx^2} = \frac{2(y^2+xy)}{(x+2y)^2}.$

(8) $\frac{y(t)}{3} = \tan\theta(t) \to \frac{dy}{dt} = 3\sec^2\theta\frac{d\theta}{dt} \;\to\; \frac{d\theta}{dt} = \frac{6}{25}$ radians/sec.

(9a) $f'(x) = \frac{-x(x+2)}{(x^2+2x+2)^2}$. Critical points: $x = 0, -2$ (note that $x^2+2x+2 \neq 0\;\forall x$).

(9b) $f(-1) = 0,\; f(0) = 1/2,\; f(1) = 2/5 \;\to\; f(x)$ has an absolute minimum at $x = -1$ with extreme value 0, $f(x)$ has a absolute maximum at $x = 0$ with extreme value $1/2$, and $f(x)$ has a local minimum at $x = 1$ with extreme value $2/5$.