Ryerson Polytechnic University

Department of
Mathematics, Physics and Computer Science

MTH 110 Term Test – October 20, 2000

Time Alloted: 1 hour 50 minutes
Examiners: R. Pyke and S. Quigley

SOLUTIONS

Instructions:

- Please have your student card ready for inspection and read the following instructions carefully.
- You are allowed one 8.5×11 inch sheet, both sides. No other aids are allowed, including no calculators.
- There is a total of six questions worth a total of 60 marks. Marks for each individual question are given in box ☐.
- This paper has a total of 9 pages, including this cover page.
- In every question show all your work. The correct answer by itself may be worth no marks. If you need more space then continue on the back of the page, directing the marker where the answer continues with a bold sign.

FOR MARKER USE ONLY

Question	Marks
1	
2	
3	
4	
5	
6	
Total	

8 **(1)** Consider the following argument form:

$$\begin{array}{l} p \to q \vee r \\ \sim q \vee p \\ \underline{q \to r} \\ \therefore p \vee r \end{array}$$

Is this a valid argument form?
Use the truth table below to explain your answer.
(Make sure you indicate the premises, conclusions and critical rows.)

Solution:

p	q	r	$p \to q \vee r$	$\sim q \vee p$	$q \to r$	$\therefore p \vee r$	
T	T	T	T	T	T	T	♠
T	T	F	T	T	F	T	
T	F	T	T	T	T	T	♠
T	F	F	F	T	T	T	
F	T	T	T	F	T	T	
F	T	F	T	F	F	F	
F	F	T	T	T	T	T	♠
F	F	F	T	T	T	F	♠

Columns 4,5 and 6 are the columns for the premises, column 7 is the column for the conclusion, and the last column has a ♠ in the (4) critical rows.

Note that the argument is <u>invalid</u> because the conclusion is false in at least one instance when all the premises are true (bottom row).

10 **(2)** Below, a set of premises and a conclusion are given. Use valid argument forms to deduce the conclusion from the premises, giving a reason for each step.

$$
\begin{array}{ll}
(a) & \sim p \rightarrow r \wedge \sim s \\
(b) & t \rightarrow s \\
(c) & u \rightarrow \sim p \\
(d) & \sim w \\
(e) & \underline{u \vee w} \\
 & \therefore \sim t \vee w
\end{array}
$$

Solution:

$$
\begin{array}{lll}
(1) & \sim w & (d) \\
 & \underline{u \vee w} & (e) \\
 & \therefore\ u & \text{by disjunctive syllogism}
\end{array}
$$

$$
\begin{array}{lll}
(2) & u \rightarrow \sim p & (c) \\
 & \underline{u \quad\;} & (1) \\
 & \therefore \sim p & \text{by modus ponens}
\end{array}
$$

$$
\begin{array}{lll}
(3) & \sim p \rightarrow r \wedge \sim s & (a) \\
 & \underline{\sim p} & (2) \\
 & \therefore r \wedge \sim s & \text{by modus ponens}
\end{array}
$$

$$
\begin{array}{lll}
(4) & \underline{r \wedge \sim s} & (3) \\
 & \therefore \sim s & \text{by conjunctive simplification}
\end{array}
$$

$$
\begin{array}{lll}
(5) & t \rightarrow s & (b) \\
 & \underline{\sim s} & (4) \\
 & \therefore \sim t & \text{by modus tollens}
\end{array}
$$

$$
\begin{array}{lll}
(6) & \underline{\sim t} & (5) \\
 & \therefore \sim t \vee w & \text{by disjunctive addition}
\end{array}
$$

(3) Consider the input/output table below:

P	Q	R	S
1	1	1	1
1	1	0	0
1	0	1	1
1	0	0	0
0	1	1	0
0	1	0	0
0	0	1	0
0	0	0	1

[4] **(a)** Construct a Boolean expression having this table as its truth table. Do not simplify this expression here.
(There is more room provided here than you need for this question.)

Solution:

$$(P \wedge Q \wedge R) \;\vee\; (P \wedge \sim Q \wedge R) \;\vee\; (\sim P \wedge \sim Q \wedge \sim R)$$

Question (3) continues on the next page →

4 **(b)** Construct a circuit having the given table as its input/output table.

Solution:

The circuit contains 4 NOT gates, 3 (triple input) AND gates, and 1 (triple input) OR gate, similar (but not exactly like) Figure 1.4.15 in the text.

Question (3) continues on the next page →

$\boxed{3}$ **(c)** Simplify the Boolean expression in part (a). Justify every step of your answer, stating the property which you are applying.

Solution:

$$
\begin{array}{lll}
 & (P \wedge Q \wedge R) \; \vee \; (P \wedge \sim Q \wedge R) \; \vee \; (\sim P \wedge \sim Q \wedge \sim R) & \\
\equiv & [(P \wedge Q \wedge R) \; \vee \; (P \wedge \sim Q \wedge R)] \; \vee \; (\sim P \wedge \sim Q \wedge \sim R) & \text{by associativity} \\
\equiv & ([(P \wedge Q) \vee (P \wedge \sim Q)] \wedge R) \; \vee \; (\sim P \wedge \sim P \wedge \sim R) & \text{by distribution} \\
\equiv & ([P \wedge (Q \vee \sim Q)] \wedge R) \; \vee \; (\sim P \wedge \sim Q \wedge \sim R) & \text{by distribution} \\
\equiv & ((P \wedge t) \wedge R) \; \vee \; (\sim P \wedge \sim Q \wedge \sim R) & \text{negation law} \\
\equiv & (P \wedge R) \; \vee \; (\sim P \wedge \sim Q \wedge \sim R) & \text{identity } P \wedge t \equiv P
\end{array}
$$

$\boxed{1}$ **(d)** Draw the simplified circuit.

Solution:

The circuit has 3 NOT gates, 2 AND gates (one has a triple input the other has a double input), and 1 (double input) OR gate.

(4) The sentence,
"The Ryerson first year computer science students are now taking their MTH110 midterm."
is of the form $\forall x \in D,\ Q(x)$.

[4] **a)** Give a precise definition of D and $Q(x)$.

Solution:

$$\begin{aligned} \text{D} &= \{\text{all Ryerson first year computer science students}\} \\ \text{or D} &= \text{the set of all Ryerson first year computer science students} \end{aligned}$$

$$\text{Q(x)} = \text{x is now taking his/her MTH110 midterm}$$

[4] **b)** As you know, the sentential form $\forall x \in D,\ Q(x)$ is equivalent to $\forall x \in C,\ P(x) \to Q(x)$ as long as C and $P(x)$ are defined carefully. Give a precise definition of C and $P(x)$ for the sentence above.

Several solutions are;

$$\begin{aligned} \text{C} &= \{\text{all Ryerson students}\} \\ \text{P(x)} &= \text{x is a first year computer science student} \end{aligned}$$

$$\begin{aligned} \text{C} &= \{\text{all first year Ryerson students}\} \\ \text{P(x)} &= \text{x is in computer science} \end{aligned}$$

$$\begin{aligned} \text{C} &= \{\text{all Ryerson computer science students}\} \\ \text{P(x)} &= \text{x is in first year} \end{aligned}$$

[4] **c)** There are a few correct answers to part b). Give an alternate correct answer.

See solution to part b).

[2 bonus] **d)** For bonus marks give a third correct answer to part b).

One more solution:

$$\begin{aligned} C &= \{\text{all students}\} \\ \text{P(x)} &= \text{x is in first year at Ryerson and studies computer science} \end{aligned}$$

(5) Consider the sentence: "Every square is above at least one triangle" in Tarski's world.

[4] **a)** Rewrite the sentence above in Tarski notation using only the symbols from the list; (,), A, E, =, #, x, y, ˜, &, |, →, ↔, above, below, square, triangle.

Solution:

$$\text{A x square(x)} \rightarrow \text{ E y triangle(y) \& above(x,y)}$$

`Note:` A x E y square(x) & triangle(y) & above(x,y) *is incorrect. (That statement would be FALSE in any world where there is a block that is not a square, even if every square in that world is above at least one triangle).*

[4] **b)** Give the negation of your answer to part a) in Tarski notation.

Solution:

$$\begin{aligned} & \text{E x square(x) \&} \sim \text{(E y triangle(y) \& above(x,y))} \\ \equiv \; & \text{E x square(x) \& A y} \sim \text{triangle(y)} \mid \sim \text{above(x,y)} \end{aligned}$$

[2] **c)** Translate the answer to part b) into English without using variable names (e.g. x,y).

Solution:

There is a square which is not above any triangle.
Note that the answer for part c) depends on what you wrote down in part b); it is not simply the negation of the English sentence at the top of the page.

(6) Write the following sentences in English without using variable names:

$\boxed{4}$ **a)** $\exists n \in \mathbf{Z}\ \forall x \in \mathbf{R},\ x > n\ \vee\ x < n$

Solution:

There is an integer such that all real numbers are strictly less than or strictly greater than that integer, i.e., some integer is either strictly above or strictly below any real number.

$\boxed{4}$ **b)** $\forall x \in \mathbf{R},\ x < 0 \rightarrow \exists y \in \mathbf{R},\ y > 0\ \wedge\ x + y = 0$

Solution:

Any negative real number has a positive real additive inverse. Or less elegantly, *For any negative real number, there is a positive real number which when added to it results in zero.*