MTH108 MIDTERM TEST Solutions

1. Like lab 1, question (1e), except there are many solutions.

   (a) Augmented matrix = $\left(\begin{array}{cccc|c} 1 & 2 & -1 & 3 & 1 \\ 0 & 1 & 2 & 1 & 1 \\ 2 & 4 & -1 & 4 & 2 \\ 1 & 2 & 0 & 1 & 1 \end{array}\right)$

   (b) RREF = $\left(\begin{array}{cccc|c} 1 & 0 & 0 & -9 & -1 \\ 0 & 1 & 0 & 5 & 1 \\ 0 & 0 & 1 & -2 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right)$

   (c) $x_1 = -1 + 9t,\ x_2 = 1 - 5t,\ x_3 = 2t,\ x_4 = t$

2. Like lab 3 question 1.

   (a) For example, $\det(A) = (3 - x)$, so A is not invertible when $x = 3$.

   (b) Unique solution: $x \neq 3$; many solutions: $x = 3$; no solutions: never.

   (c) $A^{-1} = \begin{pmatrix} -7 & -6 & 5 \\ -10 & -8 & 7 \\ -1 & -1 & 1 \end{pmatrix}$

3. Like lab 5 problem 1.

   (a) -1

   (b) -1

   (c) 64

   (d) $-1/2$

   (e) -8

   (f) 6

   (g) -1

   (h) 0

4. (a) Like exercise set 1.5 Problem 3

      i. $E_1 = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -2 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$

      ii. $E_2 = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$

iii. $E_3 = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 \end{pmatrix}$

(b) Like Lab 4 problem 2. $\det(A) = -5$

5. This is problem 2 of lab 2. Gauss-Jordan elimination gives the RREF:

$$\left(\begin{array}{ccc|c} 1 & -1 & 3 & b_1 \\ -2 & 4 & 2 & b_2 \\ -1 & 5 & 13 & b_3 \end{array}\right) \sim \cdots \sim \left(\begin{array}{ccc|l} 1 & 0 & 7 & 2b_1 + \frac{1}{2}b_2 \\ 0 & 1 & 4 & b_1 + \frac{1}{2}b_2 \\ 0 & 0 & 0 & 3b_1 + 2b_2 - b_3 \end{array}\right).$$

The condition for many solutions is $3b_1 + 2b_2 - b_3 = 0$.

6. Unique $a \neq 0, \pm 2$; many $a = 2$; none $a = 0$ or $a = -2$

7. These are like questions from lab 6 only easier.

(a) Any vector of the form $(t, t, -t)$ will do.

(b) Take $\mathbf{v} = \pm\frac{1}{\sqrt{2}}(1, 0, 1)$.

(c) $\mathbf{u} = (1, 1, 1)$.

8. Almost identical to problem 7 in chapter 11.7

(a) Any reasonable picture will do.

(b) $M = \begin{pmatrix} 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 \end{pmatrix}$

(c) Since $M^2 = \begin{pmatrix} 0 & 1 & 1 & 1 & 2 \\ 1 & 0 & 0 & 2 & 1 \\ 1 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 \\ 0 & 2 & 1 & 1 & 0 \end{pmatrix}$, we find that

$$M + M^2 = \begin{pmatrix} 0 & 2 & 2 & 2 & 2 \\ 1 & 0 & 1 & 2 & 2 \\ 1 & 1 & 0 & 2 & 1 \\ 0 & 1 & 1 & 0 & 1 \\ 1 & 2 & 1 & 2 & 0 \end{pmatrix}.$$

If we add along the rows we find that the *power* of each vertex is as follows: A = 8, B = 6, C = 5, D = 3, E = 6. The rankings are: First, A; second (tie) B and E; third, C; fourth, D.