

Ryerson Polytechnic University
Friday October 12, 2001
MTH108 MIDTERM TEST
Duration: 2 hours

NAME:

STUDENT NUMBER:

SIGNATURE:

Instructions:

- Present your solutions to the following questions in the space provided. Be clear and concise. Unreadable or sloppy solutions are not acceptable.
- Use the back side of each page if you need additional space. Indicate to the grader where your solution continues.
- This exam has 2 sections: Part A is multiple choice; Part B is short answer.
- Each multiple choice question is worth 5 points. An incorrect answer to a multiple is worth zero—no penalty for guessing.
- Each short answer question is worth 16 points.
- No aids allowed, in particular **NO CALCULATORS**.

Part A: Multiple Choice

1. The system of equations

$$\begin{aligned}7x + 4y &= b_1 \\5x + 3y &= b_2\end{aligned}$$

- (a) Sometimes has many solutions, depending on the values of b_1, b_2 .
- (b) Always has a unique solution.
- (c) Sometimes has no solutions, depending on the values of b_1, b_2 .
- (d) Both (1a) and (1c)
- (e) none of the above.

2. If B and C are both invertible 4×4 matrices then $(B + C)^2$

- (a) $= B^2 + 2BC + C^2$
- (b) $= B^2 + BC + CB + C^2$
- (c) $= B^2 + CB + BC + C^2$
- (d) Both (2a) and (2b)
- (e) Both (2b) and (2c)

3. If A and B are both $n \times n$ invertible matrices,

- (a) AB is invertible.
- (b) $A + B$ is invertible
- (c) $(AB)^{-1} = A^{-1}B^{-1}$
- (d) Both (3a) and (3b).
- (e) Both (3a) and (3c).

4. A 2×4 system of equations

- (a) May have no solutions
- (b) Always has many solutions.
- (c) Never has a unique solution.
- (d) Both (4b) and (4c).
- (e) Both (4a) and (4c).

Part B: Short Answer

1. Let $A = \begin{bmatrix} 1 & -1 & 2 \\ 1 & 0 & -1 \\ -3 & 1 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} -1 \\ 0 \\ 2 \end{bmatrix}$

- (a) Find A^{-1} .
- (b) Using *the definition* of inverse, demonstrate that your answer in (1a) is correct.
- (c) Solve $AX = B$.

2. For the augmented matrix

$$\begin{pmatrix} 1 & -2 & -7 & -1 \\ 2 & -3 & -12 & 2 \\ 1 & -1 & -5 & k \end{pmatrix},$$

- (a) Write down the corresponding system of equations.
- (b) Put this matrix into row reduced echelon form and find the value of k for which this system has many solutions.
- (c) For your choice of k in (2b) find the solution of the system in (2a).

3. If A and B are 3×3 matrices with $\det(A) = 3$ and $\det(B) = 4$. Find the following:

(a) $\det(2A)$

(b) $\det(AB^{-1})$

(c) $\det(A(B^{-1})^T)$

4. Find $\begin{vmatrix} 2 & 1 & 0 & 1 & 1 & 0 \\ 2 & 2 & 1 & 1 & 2 & 1 \\ 4 & 2 & 3 & 4 & 2 & 1 \\ 0 & 0 & 3 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 & 2 & -2 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{vmatrix}$

5. For which values of the constant a does the following system have

$$\begin{aligned}ax - y + 2z &= 2 \\3y + 2az &= -3 \\ay + 6z &= 3\end{aligned}$$

- (a) one solution
- (b) no solutions
- (c) many solutions

Justify your answers