Ryerson Polytechnic University
Monday December 10, 2001
MTH108 Final Exam
Duration: 3 hours

NAME:

STUDENT NUMBER:

SIGNATURE:

Instructions:

- Present your solutions to the following questions in the space provided. Be clear and concise. Unreadable or sloppy solutions are not acceptable.
- Use the back side of each page if you need additional space. Indicate to the grader where your solution continues.
- This exam has 2 sections: Part A is multiple choice; Part B is short answer. There are 6 multiple choice questions and 8 short answer questions. Make sure your examination copy has all questions.
- Each multiple choice question is worth 3 points. An incorrect answer to a multiple is worth zero—no penalty for guessing.
- Each short answer question is assigned a value that is written beside it.
- No aids are allowed, in particular **NO CALCULATORS**.

Part A: Multiple Choice

1. The system of equations

$$\begin{aligned} \tfrac{3}{2}x - \tfrac{2}{3}y &= b_1 \\ -9x + 4y &= b_2 \end{aligned}$$

   (a) Never has solutions.
   (b) Has no solutions when $b_1 = 1$, $b_2 = -6$.
   (c) Has no solutions when $b_1 = b_2$.
   (d) Both (1b) and (1c).
   (e) None of the above.

2. Suppose A is a 3×3 matrix with $\det(A) = -1$. Then $\det(-2A) =$

   (a) -2
   (b) 2
   (c) -8
   (d) 8
   (e) None of the above

3. Consider the plane with normal vector $\mathbf{n}$ passing through the point P_0 and the line passing through R_0 parallel to the vector $\mathbf{v}$. If $\mathbf{v} \cdot \mathbf{n} = 0$,

   (a) the line is orthogonal to the plane.
   (b) the line is parallel to the plane.
   (c) the line never intersects the plane
   (d) both (3b) and (3c)
   (e) none of the above.

4. As usual, let C_{23} denote the $(2,3)^{\text{rd}}$ cofactor of $A = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 0 & 0 & 0 & 0 \\ 4 & 3 & 2 & 1 \\ 1 & -1 & 1 & -1 \end{bmatrix}$

   (a) $C_{23} = 0$
   (b) $C_{23} = 20$
   (c) $C_{23} = -20$
   (d) $C_{23} = 5$
   (e) None of the above

5. The area of the paralellogram with vertices $(1,1)$ $(5,2)$ $(2,5)$ $(6,6)$ is

(a) 15
(b) 16
(c) 14
(d) 13
(e) None of the above

6. Let T_1 be a counterclockwise rotation by π and let T_2 be a reflection about the x-axis. Which of the following best represents the transformation obtained by first preforming T_1, and then T_2.

(a) $\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$
(b) $\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$
(c) $\begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$
(d) $\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$
(e) None of the above.

Part B: Short Answer

1. Consider the triangular system in which a and b are unspecified constants (their values do not depend on x or y or z). 3+3+3 pts.

$$\begin{aligned} x + 2y + z &= 2 \\ y - 2z &= 1 \\ az &= b \end{aligned}$$

For exactly which values of a and b does the solution set contain:

(a) No solutions?

(b) Infinitely many solutions?

(c) Exactly one solution?

2. Let $A = \begin{pmatrix} 1 & 2 & 3 \\ 1 & 3 & 4 \\ 1 & 3 & 5 \end{pmatrix}$.

5+2+3 pts.

(a) Find A^{-1}.

(b) Show that your answer in part (2a) satisfies the definition of inverse.

(c) Solve the system of equations $AX = \begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix}$

3. Given that $B = \begin{bmatrix} 2 & 3 \\ -1 & 1 \end{bmatrix}$; $A = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}$ and $|A| = 3$; Evaluate, showing your work below:

3+3+3 pts.

(a) $\begin{vmatrix} a & b & c \\ a-2d & b-2e & c-2f \\ g & h & i \end{vmatrix}$

(b) $\det(A^t(2A)^{-1})$

(c) $3I - B^2$

4. Let $\mathbf{a} = (2, 1, -1)$ and $\mathbf{u} = (3, 8, 2)$ 5+5 pts.

(a) Find the projection of $\mathbf{u}$ onto $\mathbf{a}$, i.e., $\text{proj}_a \mathbf{u}$

(b) Write $\mathbf{u}$ as a sum of two vectors, one parallel to $\mathbf{a}$ and one orthogonal to $\mathbf{a}$.

5. Consider the linear transformation $T : \mathbf{R}^2 \to \mathbf{R}^2$ which is first a rotation counter clockwise through $\frac{\pi}{4}$ followed by reflection about the line $y = x$.

4+4+4 pts.

(a) Write down the matrix for T.

(b) Write down the matrix for T^{-1}.

(c) Draw the image of the unit square in the space provided below.

6. Consider the linear transformation $T : \mathbf{R}^3 \to \mathbf{R}^3$ defined by,

$$T(x, y, z) = (3x - 2y + z, x - 4y + z, x + y)$$

4+4+4 pts.

(a) Is T one-to-one? If so, explain. If not find two distinct vectors $\mathbf{v}$ and $\mathbf{w}$ with $T(\mathbf{v}) = T(\mathbf{w}) = 0$.

(b) Is T onto? If so, explain. If not find one vector in the range of T and one vector not in the range of T.

(c) Does T have an inverse? If so, write down the formula for T^{-1}. If not, explain why.

7. Let P be the plane which contains the origin and the points $(1, 0, 1)$ and $(2, 1, 2)$. Let ℓ be the line which contains the points $(-1, 1, 0)$ and $(3, -1, 4)$.

3+3+4 pts.

(a) Write down the equation for P.

(b) Write down the equation for ℓ.

(c) Find the point of intersection of ℓ and P.

8. Let $z = -i$.

3+3+4 pts.

(a) Find z^{22}

(b) Find *all* cube roots of z, and write your answers in the form $a + ib$.

(c) Draw a carefully labeled sketch of the roots found above.