

MTH 207 Midterm Test

February 21, 2003

Name: _____

Student Number: _____

Section (circle): 001 (Delic) 002 (Filip) 003 (Pyke) 004 (Lan) 005 (Pascal) 006 (Kim)

Time allotted: 90 minutes. No aids allowed, including no calculators.

Marks for each question are indicated by []. Total marks: 55. There are **9** questions on this test.

Show all your work; answers by themselves may not be worth any marks.

[11] (1) Find the following limits if they exist. If they don't exist explain why they don't.

(a) $\lim_{x \rightarrow 0} \frac{5 + \sin(32x)}{7x^2 - x - 1}$

(b) $\lim_{x \rightarrow 8} \frac{|x - 8|}{x - 8}$

Question 1 continues →

$$(c) \lim_{x \rightarrow 0} \frac{1 - \sqrt{1 - x^2}}{x}$$

$$(d) \lim_{x \rightarrow 0} \frac{\sin 4x}{x \tan(x + \pi/4)}$$

[5] (2) Find a value of the constant c so that the function $f(x)$ is continuous for all x ;

$$f(x) = \begin{cases} 2x - c & \text{if } x \leq 1 \\ 4 - x^2 & \text{if } x > 1 \end{cases}$$

[5] (3) Use the **definition of the derivative** to find the derivative of $f(x) = \frac{1}{x+1}$.

[9] (4) In the following find $\frac{dy}{dx}$. Do not simplify your answer.

(a) $y = \tan(\sqrt{1-x})$

(b) $y = x^{-2} \sin^2(x^3)$

(c) $y = \left(\frac{\sqrt{x}}{2\sqrt{x+1}} \right)^2$

[5] **(5)** Find all the points on the curve $y = \tan x$, $-\pi/2 < x < \pi/2$ where the normal line is parallel to the line $y = -x/2$. Sketch the curve and the normals together labeling each with its equation. (Recall that the normal line at a point on a curve is the line that passes through the point and is perpendicular to the tangent line to the curve at that point.)

[5] **(6)** Consider the parametrically defined curve $x(t) = t^2 + 4t - 1$, $y(t) = t + \sin t$.

(a) Find all points (x, y) where the curve has a horizontal tangent line for t in the interval $[-2\pi, 2\pi]$.

(b) Find $\frac{d^2y}{dx^2}$ at these points.

[4] (7) Find dy/dx and d^2y/dx^2 of the implicitly defined function $xy + y^2 = 1$.

(a) dy/dx :

(b) d^2y/dx^2 :

[6] (8) A balloon rises at a rate of 2 meters per second from a point on the ground 3 meters from an observer. Find the rate of change of the angle of elevation of the balloon from the observer when the balloon is 4 meters above the ground.

[5] **(9)** Consider the function $f(x) = \frac{x+1}{x^2+2x+2}$.

(a) Find all the critical points of $f(x)$.

(b) Find the extreme values of $f(x)$ in the interval $[-1, 1]$, if there are any, and where they occur.