

MTH 207 W 2002
Midterm Test
Friday, February 22, 2002

NAME:

STUDENT NUMBER:

LECTURE SECTION:

Duration: 1 hour and 50 minutes (110 minutes)

Instructions: Answer all questions. No aids are allowed, including no calculators and no formula sheets. There are **13 questions** on **7 pages**; make sure all questions are here. Marks for each question is indicated by []. Total marks: 70

[4] (1) Let $f(x) = \frac{x^2}{1+x} + \sec(x)$. Write out the even and odd parts of f , simplifying your answer as much as possible. Verify that the even part is even and that the odd part is odd.

[3] (2) Solve for y in the equation; $\ln(2y^2 - 2) + 4 = \sin x + \ln(3^x)$.

[3] (3) Find the value of $\tan(\sec^{-1} \frac{2}{\sqrt{3}})$.

[9] (4) Evaluate the following limits.

(a)

$$\lim_{x \rightarrow 0} \frac{x^2 - 1}{2x^2 - x - 1}$$

(b)

$$\lim_{x \rightarrow 1} \frac{x^2 + 2x - 3}{x - 1}$$

(c)

$$\lim_{x \rightarrow 0} \frac{1 - \cos x}{x \sin x}$$

[4] **(5)** For what value of the constant c is the function f continuous everywhere, where f is defined by

$$f(x) = \begin{cases} cx + 1 & x \leq 3 \\ x^2 + c & x > 3 \end{cases}$$

[9] **(6)** Find the derivative of each of the following functions (you do not need to simplify your answers).

(a) $f(x) = 4\sqrt{x} + \left(1 - \frac{1}{x}\right)^2$

(b) $f(x) = \sec^2(2x^3 - x + 1)$

(c) $y = \left(\frac{2\sqrt{x}}{2\sqrt{x} + 1}\right)^{1/3}$

[5] (7) Find the equations of the lines which are tangent to the graph of $f(x) = x^3$ and parallel to the line $3x - y + 1 = 0$.

[6] (8) Find all asymptotes (including oblique, or slanted, asymptotes if there are any) of the graph of the following function;

$$y = \frac{x^3 - 4x^2}{x(x - 1)}$$

[4] **(9)** Use the Intermediate Value Theorem to show that there are at least two solutions of the following equation $x^3 + x^2 - 2x = 1$

[6] **(10)** The function $s(t) = (t^3/3) - t^2 + 1$ describes the motion of a body moving along a line with s in metres and t in seconds.

(a) Find the body's displacement and average velocity for the time interval $0 \leq t \leq 2$.

(b) When is the particle moving to the left? To the right?

(c) When is the body's velocity increasing? Decreasing?

[6] **(11)** This question concerns the curve defined implicitly by the equation $2x^3 + y^3 = 1$.

(a) Find $\frac{dy}{dx}$ at the point on the curve with x -coordinate 1.

(a) Find $\frac{d^2y}{dx^2}$ at the point on the curve with x -coordinate 1.

[5] **(12)** This equation concerns the curve defined parametrically by the equations

$$x = -\sqrt{t-1}, \quad y = \sqrt{3t+2}$$

(a) Find $\frac{dy}{dx}$ at the point $t = 5$.

(b) Are there any points on the curve where the tangent line is horizontal? If so, what are their x and y coordinates?

[6] **(13)** A ladder 5 metres long is leaning against a building. The bottom of the ladder is dragged along the ground, away from the building, at 3 metres per minute. At what rate is the angle θ between the ladder and the ground changing when the top of the ladder is 3 metres above the ground?